Solution

Department of Electrical and Electronic Engineering
Bangladesh University of Engineering and Technology

EEE 311 Digital Signal Processing-I

Class Test-1, Date: 21-3-2011, Time: 15 min

Student#: Name: Date:

1. A continuous-time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 250 Hz, 450 Hz, 1.0 kHz, 2.22 kHz, and 4.05 kHz. The signal $x_a(t)$ is sampled at a 1.5-kHz rate, and the sampled sequence is then passed through an ideal lowpass filter (LPF) with a cutoff frequency of 480 Hz, generating a continuous-time time signal $y_a(t)$. What are the frequency components present in the reconstructed signal $y_a(t)$? (10)

2. a) State with reason(s) if the following signal is periodic: (2)

   (i) $x(n) = \cos(0.6\sqrt{\pi}n + 0.2)$

   b) An analog signal $x_a(t) = 10\cos 2000\pi t + 2\sin(3000\pi t + \pi/4) + 2\sin(5000\pi t - \pi/4)$ is sampled at a rate of 4000 Hz. Find the resulting sampled signal. Dose this sampling rate cause any aliasing? (ignore this part) (1+1+6)

   I. What is the Nyquist rate for this signal?
   II. What is the folding frequency?
   III. Find the resulting sampled signal $x(n)$.

Ans. 1. $x_a(t) = A_1 \sin(2\pi F_1 t + \theta_1) + \cdots + A_5 \sin(2\pi F_5 t + \theta_5)$

$F_1 = 250$ Hz
$F_2 = 450$ Hz
} These two components will not be distored.

$F_3 = 1000$ Hz
$F_4 = 2220$ Hz
$F_5 = 4050$ Hz
} Alias frequencies of these components are

$F_3' = F_3 \pm kF_s = 1000 - 1500 = -500$ Hz.
$F_4' = 2220 - 1500 = 720$ Hz.
$F_5' = 4050 - 4500 = -450$ Hz.

$F_s = 1500$ Hz.

Folding freq $= F_s/2 = 750$ Hz.

~~The sampled signal after low pass filtering~~ $y_a(t)$ will contain ~~F_1, F_2~~ 250 Hz & 450 Hz only.

F_5' component will be aliased with F_2 and F_3 and F_4 will be filtered out by the low-pass filter.

2. (a) (i) $x(n) = \cos\left(0.6\sqrt{\pi}\, n + 0^2\right)$

Here, $\omega = 0.6\sqrt{\pi} = 2\pi f$

$$\Rightarrow f = \frac{0.6\sqrt{\pi}}{2\pi} = \frac{0.3}{\sqrt{\pi}} = \frac{3}{10\sqrt{\pi}}$$

Since π is an irrational number, $f \neq \frac{K}{N}$, the ratio of two integer numbers. Therefore, $x(n)$ is not periodic.

(b) $x_a(t) = 10\cos 2000\pi t + 2\sin\left(3000\pi t + \pi/4\right) + 2\sin\left(5000\pi t - \pi/4\right)$

$F_s = 4000$ Hz.

$$x(n) = 10\cos\left(\frac{2000}{4000}\pi n\right) + 2\sin\left(\frac{3000}{4000}\pi n + \pi/4\right) + 2\sin\left(\frac{5000}{4000}\pi n - \pi/4\right)$$

$$= 10\cos\left(\frac{\pi}{2}n\right) + 2\sin\left(\frac{3}{4}\pi n + \pi/4\right) + 2\sin\left(\frac{5}{4}\pi n - \pi/4\right)$$

$$= 10\cos\left(\frac{\pi}{2}n\right) + 2\sin\left(\frac{3}{4}\pi n + \pi/4\right) + 2\sin\left\{\left(\frac{5\pi}{4} - 2\pi\right)n - \pi/4\right\}$$

$$= 10\cos\left(\frac{\pi}{2}n\right) + 2\sin\left(\frac{3}{4}\pi n + \pi/4\right) - 2\sin\left(\frac{3\pi}{4}n + \pi/4\right)$$

$$= 10\cos\left(\frac{\pi}{2}n\right)$$

Yes, this F_s cause aliasing. $F_3 = 2500$ Hz is aliased with $F_2 = 1500$ Hz.

I. Nyquist rate $= 5000$ Hz.

II. Folding freq $= \frac{F_s}{2} = 2000$ Hz.

III: $x(n)$ is determined above.