Solution CT2

Bangladesh University of Engineering and Technology

Department of Electrical and Electronic Engineering

EEE 311: Digital Signal Processing

April 2011 Quiz # 2 Marks: 20 Time: 25 minutes

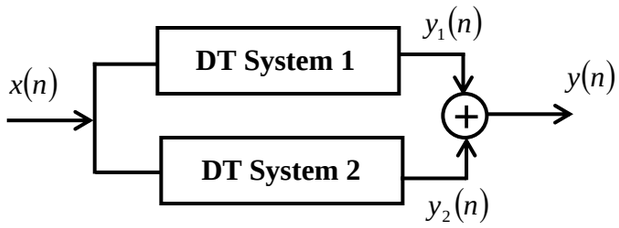
Question 1: Two LTI systems are connected in parallel as shown in figure below. The input-output relationships of the DT systems are also given.

DT System 1:

$$y_1(n)=\sum_{k=n-2}^{n}x(k)$$

DT System 2:

$$y_2(n)=-\sum_{k=n-7}^{n-4}x(k)$$



(a) Determine and plot the impulse responses $h_1(n)$ and $h_2(n)$ of the DT Systems 1 and 2, respectively.
(b) Determine whether the DT Systems 1 and 2 are stable. Justify your answer.
(c) Determine whether the DT Systems 1 and 2 are causal. Justify your answer.
(d) Determine and plot the impulse response of the overall system.
(e) Determine the outputs $y(n)$ of the overall system, when the inputs are:

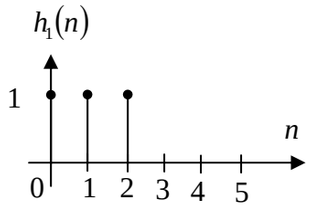
(i) $x(n)=u(n)-u(n-3)$

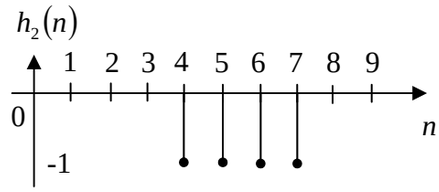
(ii) $x(n)=(-1)^n[u(n)-u(n-2)]$

Hint: Note that $\delta(n)=u(n)-u(n-1)$

Solution:

(a) $h_1(n)=\delta(n)+\delta(n-1)+\delta(n-2)$ and $h_2(n)=-\delta(n-4)-\delta(n-5)-\delta(n-6)-\delta(n-7)$



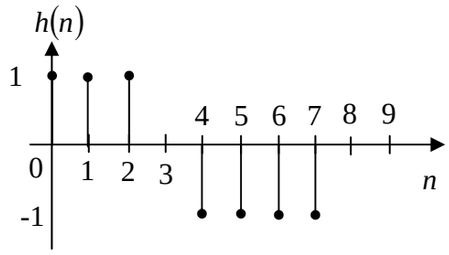


(b) Since $\sum_{n=-\infty}^{\infty}|h_1(n)|=3<\infty$ and $\sum_{n=-\infty}^{\infty}|h_2(n)|=4<\infty$ the systems are stable.

(c) Since $h_1(n)=0$ for $n<0$ and $h_2(n)=0$ for $n<0$ the systems are causal.

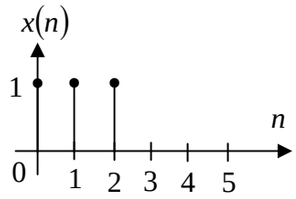
(d) Impulse response of the overall system is

$$h(n)=h_1(n)+h_2(n)=\delta(n)+\delta(n-1)+\delta(n-2)-\delta(n-4)-\delta(n-5)-\delta(n-6)-\delta(n-7)$$



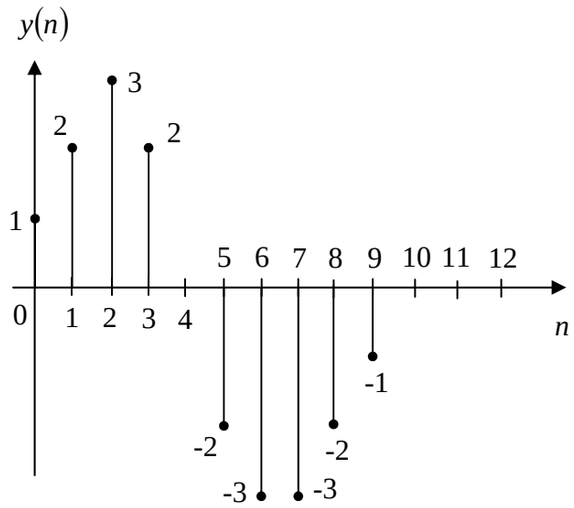
Solution CT2

(e) (i) Here $x(n)=u(n)-u(n-3)=\delta(n)+\delta(n-1)+\delta(n-2)$

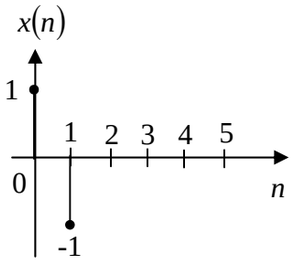


Hence

$$y(n)=h(n)\otimes x(n)$$
$$=\delta(n)+\delta(n-1)+\delta(n-2)-\delta(n-4)-\delta(n-5)-\delta(n-6)-\delta(n-7)+$$
$$\delta(n-1)+\delta(n-2)+\delta(n-3)-\delta(n-5)-\delta(n-6)-\delta(n-7)-\delta(n-8)+$$
$$\delta(n-2)+\delta(n-3)+\delta(n-4)-\delta(n-6)-\delta(n-7)-\delta(n-8)-\delta(n-9)$$
$$=\delta(n)+2\delta(n-1)+3\delta(n-2)+2\delta(n-3)-2\delta(n-5)-3\delta(n-6)-3\delta(n-7)-2\delta(n-8)-\delta(n-9)$$



(ii) Here $x(n)=(-1)^n[u(n)-u(n-2)]=\delta(n)-\delta(n-1)$



Hence

$$y(n)=h(n)\otimes x(n)$$
$$=\delta(n)+\delta(n-1)+\delta(n-2)-\delta(n-4)-\delta(n-5)-\delta(n-6)-\delta(n-7)+$$
$$-\delta(n-1)-\delta(n-2)-\delta(n-3)+\delta(n-5)+\delta(n-6)+\delta(n-7)+\delta(n-8)$$
$$=\delta(n)-\delta(n-3)-\delta(n-4)+\delta(n-8)$$