Solution

Dept. of EEE, BUET

EEE 311: Class Test 3 **9 May 2011** **Marks: 20** **Time: 25 minutes**

STN: **Name:**

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1. Determine the inverse z-transform of $H(z)=\dfrac{z^2}{z^3-2.5z^2+2z-0.5}$ if
(a) ROC: $|z|>1$ (b) ROC: $|z|<0.5$ (c) ROC: $0.5<|z|<1$

2. Determine the zero-state response of the system $y(n)=0.5y(n-1)+4x(n)+3x(n-1)$ to the input $x(n)=e^{j\omega_0 n}u(n)$. What is the steady-state response of the system?

$\boxed{1.}$ $$H(z)=\frac{z^2}{z^3-\frac{5}{2}z^2+2z-\frac{1}{2}} \Rightarrow \frac{H(z)}{z}=\frac{z}{(z-1)^2(z-\frac{1}{2})}$$

$$\Rightarrow \frac{H(z)}{z}=\frac{A}{z-1}+\frac{B}{(z-1)^2}+\frac{C}{z-\frac{1}{2}}=\frac{z}{(z-1)^2(z-\frac{1}{2})}$$

$$\therefore C=\left.\frac{z}{(z-1)^2}\right|_{z=\frac{1}{2}}=2;\quad B=\left.\frac{z}{z-\frac{1}{2}}\right|_{z=1}=2$$

$$A=\left.\frac{d}{dz}\left[\frac{z}{z-\frac{1}{2}}\right]\right|_{z=1}=\left.\frac{(z-\frac{1}{2})-z}{(z-\frac{1}{2})^2}\right|_{z=1}=\frac{-\frac{1}{2}}{(-\frac{1}{2})^2}=-2$$

$$\therefore \frac{H(z)}{z}=\frac{2}{z-\frac{1}{2}}-\frac{2}{z-1}+\frac{2}{(z-1)^2}$$

$$\therefore H(z)=\frac{2}{1-\frac{1}{2}z^{-1}}-\frac{2}{1-z^{-1}}+\frac{2z^{-1}}{(1-z^{-1})^2}$$

(a) ROC: $|z|>1 \Rightarrow h(n)=2\left(\frac{1}{2}\right)^n u(n)-2u(n)+2nu(n)$

(b) ROC: $|z|<0.5 \Rightarrow h(n)=-2\left(\frac{1}{2}\right)^n u(-n-1)+2u(-n-1)-2nu(-n-1)$

(c) ROC: $0.5<|z|<1 \Rightarrow h(n)=2\left(\frac{1}{2}\right)^n u(n)+2u(-n-1)-2nu(-n-1)$

$\boxed{2}$

Given. $y(n) = \frac{1}{2} y(n-1) + 4x(n) + 3x(n-1)$

$$\Rightarrow Y(z) = \frac{1}{2} z^{-1} Y(z) + 4X(z) - 3X(z) z^{-1}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{4 + 3z^{-1}}{1 - \frac{1}{2} z^{-1}}$$

Since, $x(n) = e^{j\omega_0 n} u(n) \quad \therefore X(z) = \frac{1}{1 - e^{j\omega_0} z^{-1}}$

$$Y(z) = \frac{4 + 3z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - e^{j\omega_0} z^{-1}\right)}$$

$$\frac{Y(z)}{z} = \frac{4z + 3}{\left(z - \frac{1}{2}\right)\left(z - e^{j\omega_0}\right)} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z - e^{j\omega_0}}$$

$$\therefore A = \left.\frac{4z+3}{z - e^{j\omega_0}}\right|_{z=\frac{1}{2}} = \frac{5}{\frac{1}{2} - e^{j\omega_0}} \quad \text{and} \quad B = \left.\frac{4z+3}{z - 1/2}\right|_{z = e^{j\omega_0}}$$

$$= \frac{4e^{j\omega_0} + 3}{e^{j\omega_0} - 1/2}$$

$$\therefore Y(z) = \frac{5}{\frac{1}{2} - e^{j\omega_0}} \cdot \frac{1}{1 - \frac{1}{2} z^{-1}} + \frac{4e^{j\omega_0} + 3}{e^{j\omega_0} - \frac{1}{2}} \cdot \frac{1}{1 - e^{j\omega_0} z^{-1}}$$

$$\therefore y(n) = \left[\frac{5}{\frac{1}{2} - e^{j\omega_0}} \cdot \left(\frac{1}{2}\right)^n + \frac{4e^{j\omega_0} + 3}{e^{j\omega_0} - \frac{1}{2}} \cdot e^{j\omega_0 n}\right] u(n)$$

Hence, the steady state response is

$$y_{ss}(n) = \lim_{n \to \infty} y(n) = \frac{4e^{j\omega_0} + 3}{e^{j\omega_0} - \frac{1}{2}} \cdot e^{j\omega_0 n}$$