Solution

Bangladesh University of Engineering and Technology

Department of Electrical and Electronic Engineering

EEE 311: Digital Signal Processing I

May 2011 Quiz # 4 Marks: 20 Time: 25 minutes

Name: ---------------- (First) ---------------- (Last) **ID#** ----------------

If otherwise stated, the symbols have their usual meanings.

Q1: Consider a discrete-time LTI system is described by the following difference equation

$$y(n) = x(n) - x(n-4)$$

(a) Plot the magnitude and phase of the frequency response function, i.e., $H(\omega)$, which is obtained from the impulse response $h(n)$ of the system for $-\pi \le \omega \le \pi$.

(b) A discrete-time input signal $x(n)$ is obtained by sampling an analog signal $x_a(t) = T_s\left[\frac{\sin(10t)}{\pi t}\right]$ with a sampling rate $T_s = \frac{2\pi}{40}$. Plot the magnitude spectrum of the DTFT of the sampled signal $x(n)$ denoted as $|X(\omega)|$.

(c) Plot the magnitude spectrum of the DTFT of the corresponding sampled output signal $y(n)$ denoted as $|Y(\omega)|$.

(d) Determine the numerical value of $\sum_{n=-\infty}^{n=+\infty} |y(n)|^2$ using Parseval's relation.

Hint: You may or may not need the following relations

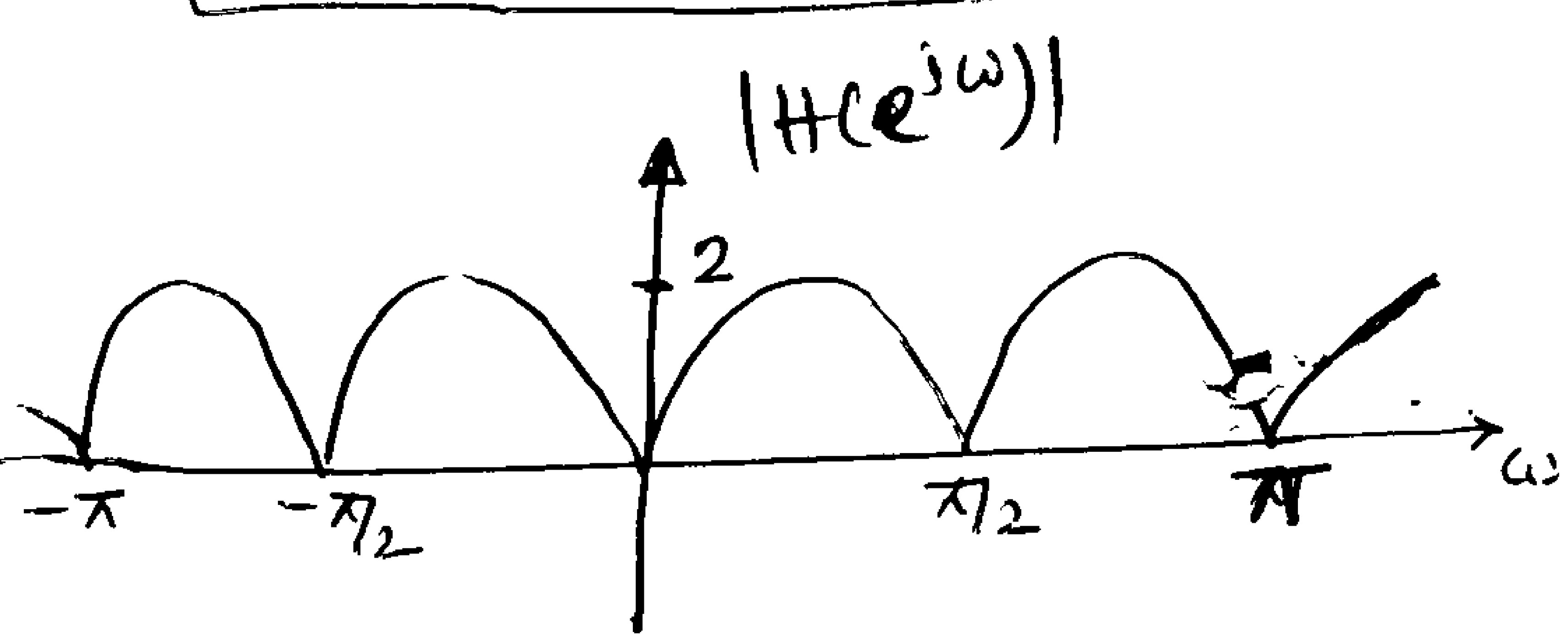
DT Domain	DTFT Domain
$x(n) = \begin{cases} \frac{\omega_c}{\pi} & n = 0 \\ \frac{\sin(\omega_c n)}{\pi n} & n \ne 0 \end{cases}$	$X(\omega) = \begin{cases} 1 & \lvert\omega\rvert \le \omega_c \\ 0 & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
$\sum_{n=-\infty}^{n=+\infty} \lvert x(n)\rvert^2$	$\frac{1}{2\pi}\int_{2\pi} \lvert X(\omega)\rvert^2 d\omega$

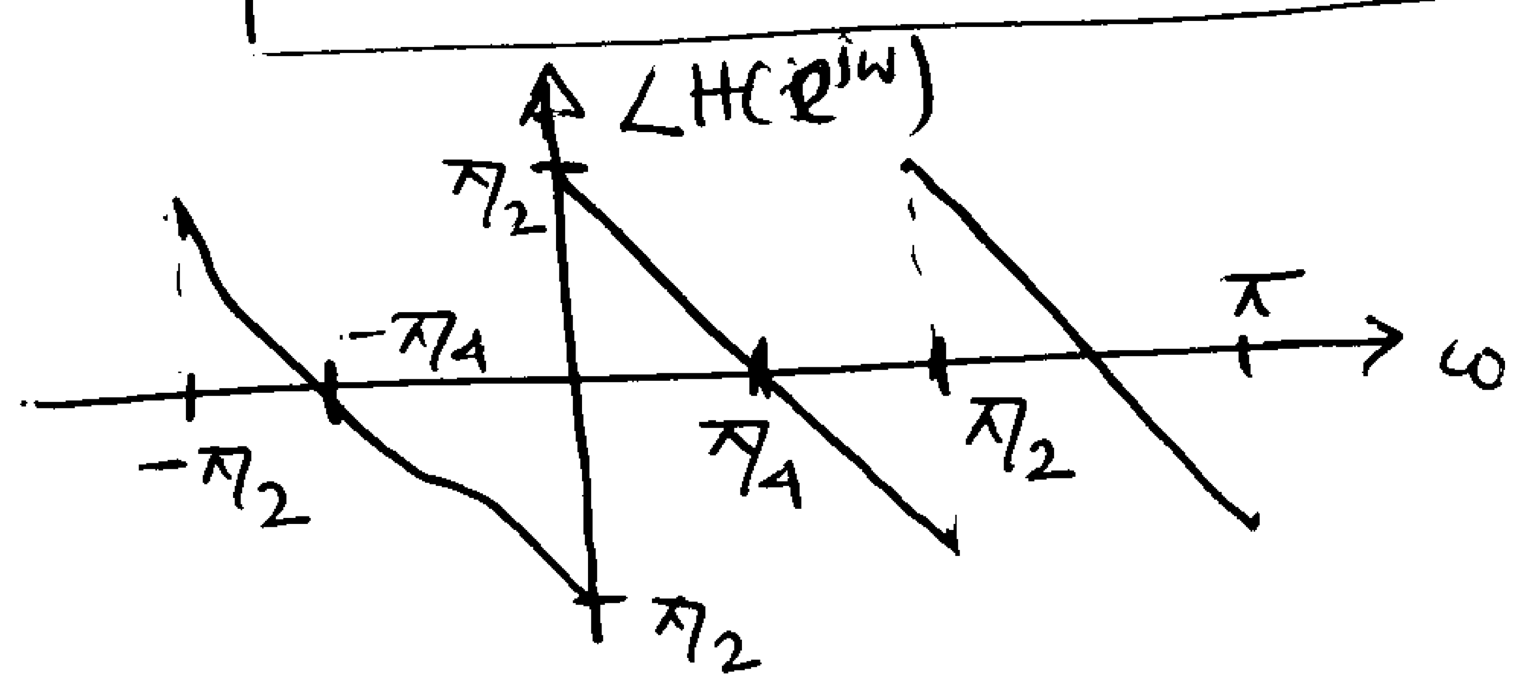
(a) Taking DTFT of the difference equation:

$$Y(e^{j\omega}) = X(e^{j\omega}) - X(e^{j\omega})e^{-j\omega 4} = X(e^{j\omega})(1 - e^{-j\omega 4})$$

$$\therefore H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = e^{-j\omega 2}(e^{j\omega 2} - e^{-j\omega 2}) = 2j e^{-j\omega 2}\sin(2\omega)$$

$$|H(e^{j\omega})| = 2\sin(2\omega)$$ and $$\angle H(e^{j\omega}) = \begin{cases} \frac{\pi}{2} - 2\omega, & \sin(2\omega) > 0 \\ \frac{\pi}{2} - 2\omega \pm \pi, & \sin(2\omega) < 0 \end{cases}$$

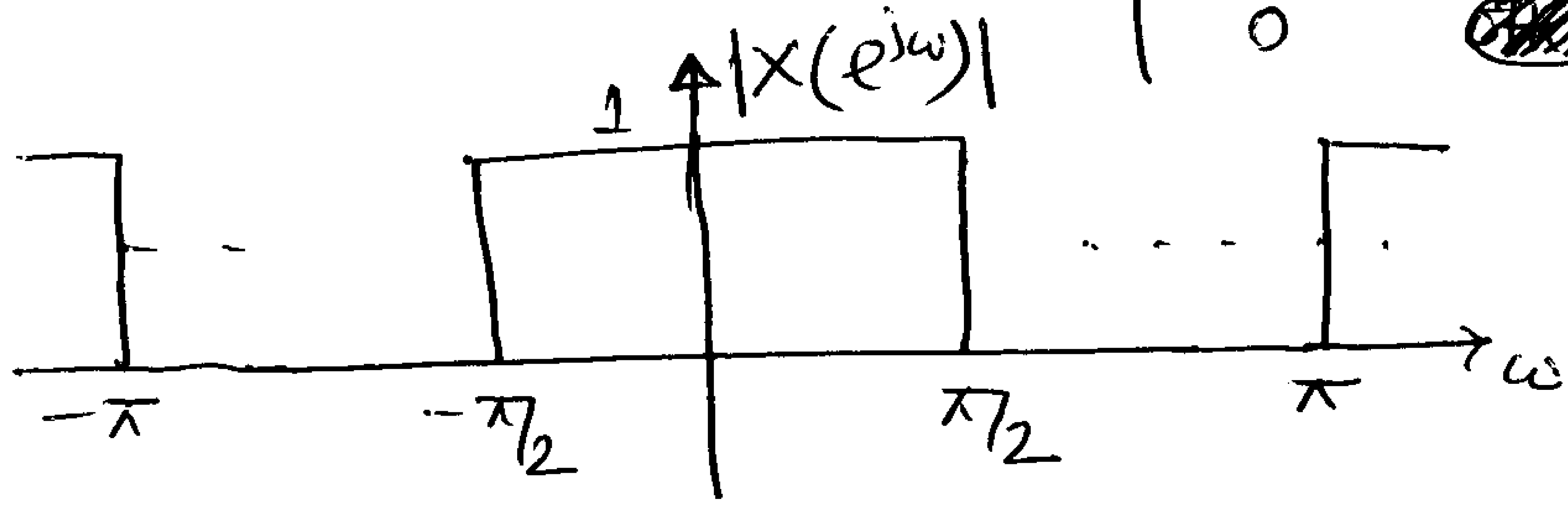




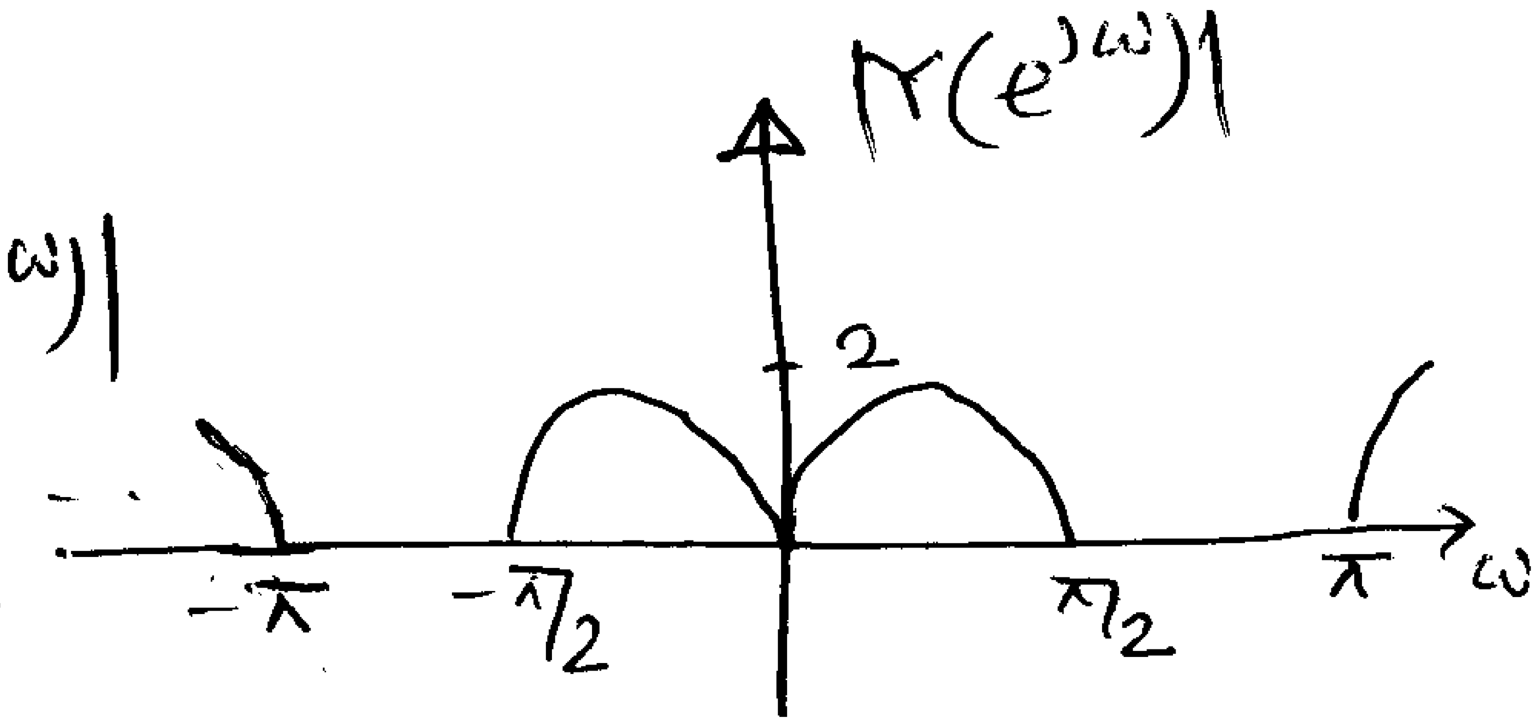
(b) $x(n) = x_a(nT_s) = T_s\left[\dfrac{\sin(10nT_s)}{\pi n T_s}\right] = \dfrac{\sin\left(10 \cdot n \cdot \frac{2\pi}{40}\right)}{\pi n}$

$= \dfrac{\sin \frac{\pi}{2} \cdot n}{\pi n}$

from table, $|X(e^{j\omega})| = \begin{cases} 1, & \text{[redacted]} \quad 0 \le \omega \le \pi/2 \\ 0 & \text{[redacted]} \quad \pi/2 \le \omega \le \pi \end{cases}$



(c) Since, [redacted] $|Y(e^{j\omega})| = |X(e^{j\omega})|\,|H(e^{j\omega})|$



(d) $\displaystyle\sum_{n=-\infty}^{\infty} |y(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 4\sin^2(2\omega)\, d\omega = \frac{1}{2\pi}\cdot 4 \cdot 2 \int_0^{\pi/2} \sin^2(2\omega)\, d\omega$

$\displaystyle = \frac{2}{\pi}\int_0^{\pi/2} (1 - \cos 4\omega)\, d\omega = \frac{2}{\pi}\left[\omega\Big|_0^{\pi/2} - \frac{1}{4}\sin 4\omega\Big|_0^{\pi/2}\right]$

$\displaystyle = \frac{2}{\pi}\left[\frac{\pi}{2} - 0\right] = 1.$ Ans